pressure distribution, circulation and lift can be predicted accurately for small change in mean velocity ($0.7 < VR < 1.25$). The expression for vortex distribution, derived in this paper satisfies the kinematic and Kutta conditions exactly. It is found that the ratio of lift coefficients in diverging or converging flow and the two-dimensional value varies linearly with VR, i.e.,

$$C_L/C_{L2d} \simeq VR$$

In a converging flow, the circulation and lift increases with increase in velocity ratio (outlet velocity/inlet velocity). There is also considerable change in airfoil pressure distribution, its effect being dominant on the suction surface. In a diverging flow, both the circulation and lift decrease with decrease in velocity ratio. The effect of change in mean velocity on pressure distribution is likely to be dominant on the pressure surface. These changes in the airfoil pressure distribution affect the viscid characteristics of the airfoil, thus adversely affecting the cavitation characteristics of a hydrofoil or pump blade, stall characteristics of a wing, pressure rise characteristics of a fan or compressor blade row. Incorporation of these effects is, thus, a practical necessity.

The pressure distribution measured at midspan and at $Z =$ 3 in. are found to be identical, thus confirming the validity of the quasi two-dimensional approach taken in this paper. The theory is valid exactly at the midspan of the airfoil, where the spanwise velocity is zero, and becomes progressively inaccurate near the converging or diverging walls, where the spanwise velocities cannot be neglected.

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Methodology for Structural Optimization of STOL Aircraft Vertical Stabilizers

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A method is described for selecting the optimum vertical surface configuration for STOL transport configurations, based upon structural weight and performance requirements. A minimization technique, using the Fiacco-McCormick penalty function[1] is used to obtain a solution based upon minimization of an objective function. Design loads for the vertical surface are considered to be defined by the requirement for trim under an engine failure condition. Since structural weight is configuration sensitive, the optimum surface is defined by this condition. Variables include maximum surface deflection and control surface chord ratios. Structural strength requirements are established for a range of configurations typical of STOL aircraft designs. Structural weight is defined in terms of applied load, stabilizer configuration and relevant design parameters. This relation defines an objective function which is minimized in determining the optimum stabilizer configuration based upon structural weight. The system derived is solved using the SLUMT algorithm of Fiacco and McCormick with the Powell direct search technique for constrained nonlinear optimization.[2]

Nomenclature

AR_e = vertical stabilizer effective aspect ratio
AR_v = vertical stabilizer aspect ratio
b = span, ft
c_R = vertical stabilizer root chord, ft
c_T = vertical stabilizer tip chord, ft
c_W = wing mean chord, ft
$C_{L\max}$ = airplane maximum lift coefficient
$C_{n\beta}$ = yawing moment derivative per degree sideslip
$C_{n\delta}$ = yawing moment derivative per degree control surface deflection
Cy_β = side force coefficient per degree sideslip
Cy_δ = side force coefficient per degree control surface deflection
l_v = tail length, ft
q = dynamic pressure, psf
P_{YVT} = total aerodynamic load on vertical stabilizer, lb
S = area, sq. ft
$(T-D)\bar{y}$ = yawing moment due to loss of engine, ft-lb
W = weight, lb

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Index categories: Optimal Structural Design; Aircraft Configuration Design.

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χ/c = hinge line location
β = angle of sideslip, degrees
δ = deflection, degrees
η_H = horizontal tail position
Λ = sweepback angle of 50% chord line, deg
λ = taper ratio

Subscripts

A = complete airplane
$A-T$ = airplane less vertical stabilizer
S = flying tail
R = rudder
Vc = varicam
v, VT = vertical stabilizer
W = wing

Introduction

THE high thrust/low-takeoff speed combinations typical of short takeoff and landing (STOL) aircraft operations impose large directional control requirements with wing-mounted, multiengined configurations. This directional control capability greatly exceeds that necessary for normal operations. As a result, the structural requirements imposed under these loadings can produce large weight penalties in the vertical stabilizer and aft fuselage. In multiengined airplanes, failure of a single engine will cause a high yawing moment due to the drag of the failed engine and the thrust of the opposing engine.

The requirement for trim of the aircraft under this induced moment is accomplished using rudder deflection to the maximum pilot effort. Since available rudder control is generally insufficient at takeoff velocities to trim the directional moment, the airplane will sideslip. The increased rudder hinge moment due to sideslip may cause rudder hinge moments beyond the pilot's strength capability unless the pilot's efforts are supplemented with a powered system. This requirement for trim with a failed engine generally produces design loads for the vertical control surface. The required stabilizing moment for this condition will be developed by the aerodynamic force acting on the vertical surface. A high degree of stabilizer effectiveness is required due to the low dynamic pressure associated with STOL takeoff speeds. Since structural weight will be sensitive to the configuration employed, the optimum vertical surface may be defined by the engine out requirement. The basic configurations considered are shown in Fig. 1

The flying tail is an all movable surface with hinge line at the quarter-chord. Provision for a rudder is included where the increased effectiveness is required. The "varicam" has a double control surface which effectively provides a variable camber influence.

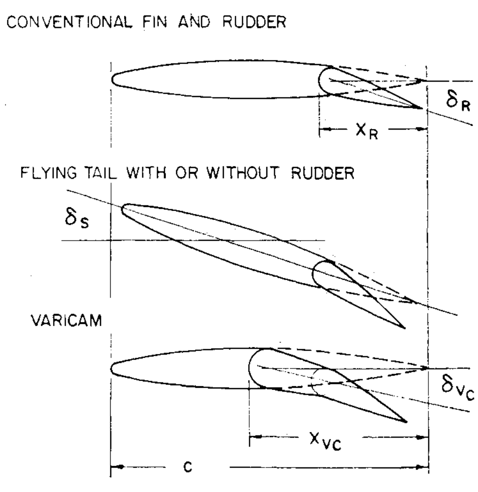

Fig. 1 Configuration definition.

Approach

Required vertical stabilizer size is established to provide a sufficient level of directional stability to overcome the destabilizing influence of the fuselage. Variables employed to represent the airplane configuration under consideration are related to vertical tail structural requirements through definition of pertinent aerodynamic and geometric characteristics.

Configuration related variables include wing geometry, tail length, airplane less vertical stabilizer weight and aerodynamic characteristics, yawing moment due to loss of an outboard engine, vertical stabilizer sweep and taper ratio. These factors represent fixed variables which define the configuration for which the optimization procedure is implemented.

The vertical stabilizer size necessary to achieve the desired directional stability level is established using the configuration related data with appropriate relations based upon available aerodynamic data. After the required vertical stabilizer geometry has been established, total airplane stability derivatives may be determined. Factors involved in this determination define vertical stabilizer geometry, directional aerodynamic characteristics, angle of sideslip, dynamic pressure, and vertical stabilizer aerodynamic loading. These dependent variables are defined by input variables and appropriate interrelationships. Dynamic pressure is determined from stall characteristics of the airplane. The value employed is the minimum dynamic pressure at which STOL operations may be effected. ($1.10 V_{stall}$ for the example included).

Criteria

Consistent with usual preliminary design practice, a simplified representation of airplane response to engine failure is effected by assuming that yawing moment and maximum corrective control surface response are developed instantaneously with the failure. A 50% increment is added to the steady-state sideslip angle required for trim to account for the dynamic overswing that occurs in such a maneuver, thus developing the maximum load which the vertical stabilizer is expected to sustain.

Since the initial size of the vertical surface is based on a cruise stability criterion, development of sufficient aerodynamic load to balance an engine failure condition at low speeds may require yaw angles well beyond vertical tail stall. Where a requirement for angle of sideslip greater than 25° (typical maximum sideslip angle for vertical stabilizer surface stall) is indicated, a larger tail size becomes necessary to provide the required side load for trim. This revised surface requirement is used with the necessary load in determination of structural weight of the stabilizer.

Vertical stabilizer weight is a function of applied load, configuration and structural design parameters:

$$\text{WEIGHT} = f(P_{YVT}, \eta_H, b_v, c_R, c_T, \Lambda_v, (X/C)_R, (X/C)_v) \quad (1)$$

Structural parameters involved in this determination include thickness ratios, structural properties of materials, minimum allowable gages, weight densities, unit weights of leading and trailing edges, and horizontal tail geometry. Structural weight is determined using a digital computer program in which cross-sectional area requirements are established at incremental tail stations using applied loads, design and material data. The interval requirements are integrated into the total vertical tail weight. Vertical tail load is assumed to be uniformly distributed over the exposed area. Geometry and material data, in conjunction with the loading, permits calculation of required cross-sectional areas. The cross-sectional area at the fuselage intersection is assumed constant through the fuselage and tail station zero. Rudder and other secondary structure weights are determined using their respective unit weight and area function.

The study contained herein is limited to conventional skin-stringer construction. The weight determination is based on ultimate applied load equal to $1.5P_{YVT}$ where

$$P_{YVT} = qS_v(C_{Y_{\beta VT}}(\beta + \delta_s) + C_{Y\delta_R}\delta_R + C_{Y\delta_V}\delta_V) \quad (2)$$

$$\beta = \frac{1}{C_{n\beta}}\left[\frac{(T-D)\bar{y}}{qS_wb_w} - C_{n\delta_R}\delta_R - C_{n\delta_v}\delta_v - C_{n\delta_s}\delta_s\right] \quad (3)$$

and other factors obtained as described previously or input within the decision variables, which include hinge lines and maximum deflections of control surfaces, and horizontal tail position.

These factors define the vertical stabilizer configuration and are varied within the search routine to determine the combination of values compatible with minimization of the objective function presented in Eq. (1). The objective of the program developed herein is definition of a vertical stabilizer configuration consistent with a minimum value of this factor using the minimization technique of Fiacco and McCormick.[1]

Process

The system derived using the relations and parameters previously defined is capable of defining a minimum weight configuration through the process of trial and error. Alternately, consistent changes into the system developed may be introduced and the optimum configuration may be selected using interpolation or extrapolation techniques. A more direct solution to find the optimum configuration for least structural weight may be effected using the process outlined in the following section. Solution of the system defined in preceding sections is effected using the CASINO algorithm[1] and Powell's direct search technique[4] for constrained nonlinear optimization.

The Slacked Unconstrained Minimization Technique (SLUMT) provides an algorithm for solution of the problem to minimize a convex function $f(x)$ subject to a number of constraints, represented by concave functions, $g_i(x) \geq 0$, $i = 1, \ldots, m$ by converting these constraints into the form

$$\left.\begin{aligned} g_i(X) - t_i &= 0 \\ t_i &> 0 \end{aligned}\right\} i = 1, \ldots, m \quad (4)$$

Difficulties encountered due to the nonlinear characteristics of these relations are alleviated by transforming the objective function (1) to a penalty function

$$P(X, t, r_k) = f(X) + r_k^{-1} \sum (g_i(X) - t_i)^2 \quad (5)$$

and the search procedure developed by Powell[4] employed to minimize this function in x, non-negative t, for an arbitrary $r_1 > 0$. From this minimum, r is reduced and the function again minimized. It has been proven that appropriate values of x exist for every $r_k > 0$, and converge to solutions of the minimization problem as $r_k \dashrightarrow 0$.[1] Powell's search technique is designed to find values of n parameters $x_1, \ldots, x_n$ to minimize the value of some function of these parameters, $f(x) = P(x, t, r_k)$. The basic procedure is repeated until a global minimum for the function $f(x)$ is determined.

The procedure starts with selection of values for the decision variables previously defined for a "best" approximation of the minimum to the objective function. WEIGHT $= P(x, t, r_k)$ where x are the fixed, dependent and independent variables included in the objective function definition as shown in Eqs. (1) and (2). The functions $g_i(x)$ define restraints upon the relations involving the decision variables

$$\delta_{S\max} \leq \delta_{R\max}, \quad \delta_{R\max} \geq \delta_{Vc\max}, \quad 0 \leq \eta_H \leq 1.0,$$
$$0 \leq (x/C)_R \leq 0.5, \quad 0 \leq (x/C)_{Vc} \leq 0.5, \quad (x/C)_R \leq (x/C)_{Vc}$$

Conversion into the SLUMT format produces the constraint functions:

$$C(1) = \delta_{S\max} - \delta_{R\max}, \quad C(2) = \delta_{R\max} - \delta_{Vc\max},$$
$$C(3) = \eta_H, \quad C(4) = 1 - \eta_H,$$
$$C(5) = (x/C)_R, \quad C(6) = 0.5 - (x/C)_R,$$
$$C(7) = (x/C)_{Vc}, \quad C(8) = 0.5 - (x/C)_{Vc},$$
$$C(9) = (x/C)_{Vc} - (x/C)_R$$

The initial point x_o selected is a feasible point in the constrained problem. n linearly independent direction vectors are defined and set equal to the coordinate directions so that each variable is changed individually by some assigned step size. The point at which the objective function is minimized is selected as the initial point for the next move. Where no further reduction in the objective function is obtained, a new direction vector is defined and employed in establishing a new minimum point. The iterative process consists of testing, moving to a new starting point or selecting a new direction vector to determine a new minimum point until the iterations reach some selected number for each response, or until the step sizes decrease below an assigned limit. At this point the desired value of the objective function and the associated decision (independent) variables are obtained.

Configuration Representation

Application of the process discussed in preceding section to a typical STOL transport configuration illustrates the type of data required to define applicable stability criteria. Directional stability requirements are highly sensitive to the airplane configuration under development. Where some knowledge regarding the variation of $C_{n\beta}$ with airplane size exists, improved results may be obtained using nonlinear characteristics. For the illustrative example discussed in the following portion of the paper, simplified relations based upon linear variations of factors are employed.

Data for existing transport configurations[3] are used to provide a guide to determination of vertical tail volume for directional stability based on the requirement that the vertical stabilizer overcome the destabilizing effect of the fuselage. Although the factors provided are derived from data based on conventional aircraft, indicated tail volumes will provide a guide to stabilizer sizes necessary for STOL configurations based on cruise stability criteria. A larger vertical surface area may be required to provide effective directional control at the low speeds where STOL configurations operate. The range of stabilizer sizes included in the investigation allow for this requirement.

A desirable level of directional stability is obtained from the data shown on Fig. 2

$$C_{n\beta} = 10^{-3}(K_1 W/S_w b_w) \quad (6)$$

The ratio K_1 is a function of the desired level of stability, airplane size, distribution of mass, and configuration. Values of K_1 for contemporary transport aircraft range from 2.3 (C-130, C-141, DC-8) to 3.1 (C-5, B720, L-1011). No particular influence of horizontal tail location upon the value of this factor is evident in the data.

During normal cruise operation, a STOL transport will have directional stability requirements identical to those for an equivalent, conventional airplane configuration. It is at the very low-speed range for takeoff and landing operations that an additional control requirement exists for STOL configurations. Dynamic pressure is reduced as a function of the square of the airspeed; accordingly, tail effectiveness decreases rapidly with airspeed irrespective of stabilizer size. The result is that stability augmentation becomes necessary at lower speeds with increases in stabilizer area contributing little to available stability. The maximum value of K_1 from contemporary aircraft

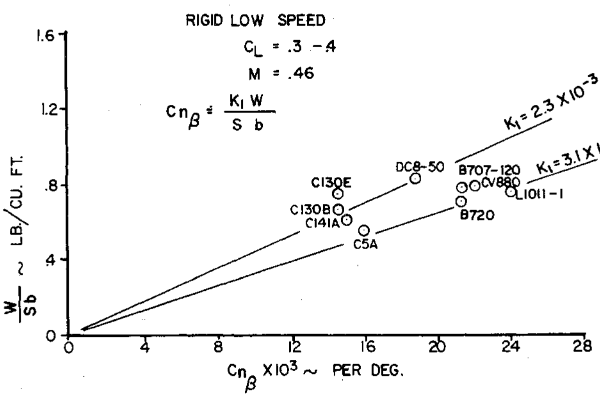

Fig. 2 Directional stability requirements.

data is retained to provide the larger stabilizer size that is required for STOL airplanes. The directional stability requirement is then

$$C_{n_\beta} = 0.0031W/S_w b_w \tag{7}$$

The effectiveness of vertical tail volume in providing directional stability is derived from available relations for jet transports (8) as shown in Fig. 3

$$\bar{V}_v = K_2 C_{n_\beta} \tag{8}$$

K_2 varies from 26 (DC-8, B707, B720, CV880, L-1011) to 38 (C-141, C-130, B727) for the general range of jet transports and is equal to 50 for the C-5A. Since the tail volume requirement for a STOL configuration will be higher than for a conventional transport as discussed in the preceding paragraph, a value of $K_2 = 40$ is selected for the present study. (The larger factors representative of a C-5 are not considered typical for STOL configurations.) The required vertical stabilizer volume is then defined by combining Eqs. (7) and (8)

$$\bar{V}_v = 0.124W/S_w b_w \tag{9}$$

but

$$\bar{V}_v = \frac{S_v l_v}{S_w b_w} \tag{10}$$

so that required vertical stabilizer size may be expressed as

$$S_v = 0.124(W/l_v) \tag{11}$$

Establishment of the stabilizer shape is based on the requirement imposed by Eq. (7). The side force coefficient is directly related to stabilizer aspect ratio so that the required shape may be determined knowing the desired directional stability. The relationship between side force effectiveness and aspect ratio

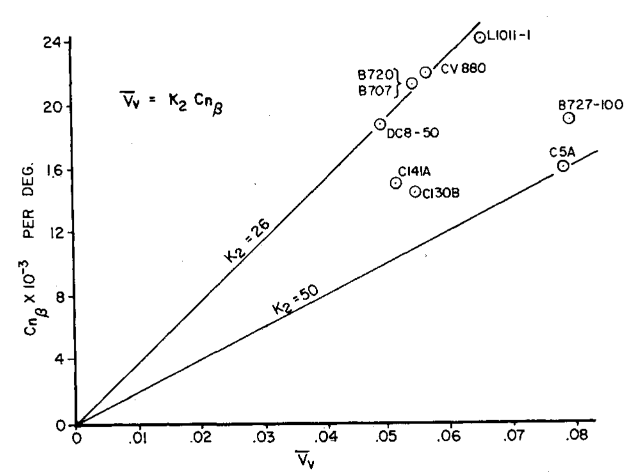

Fig. 3 Vertical stabilizer volume requirement.

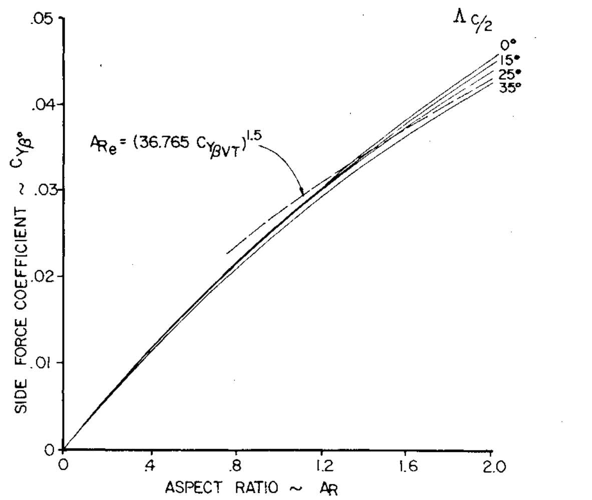

Fig. 4 Variation of $C_{Y_{\beta_{VT}}}$ with aspect ratio in incompressible flow.

is shown on Fig. 4 (from Ref. 3). The effective aspect ratio of the vertical stabilizer will differ from the geometrical aspect ratio due to the end plate effect of the horizontal tail surface. The variation as a function of horizontal stabilizer location is shown on Fig. 5 (from Ref. 4).

Control surface effectiveness is established as a function of the ratio of control surface chord to stabilizer chord as shown in Fig. 6 (from Ref. 5). For the full span control surfaces

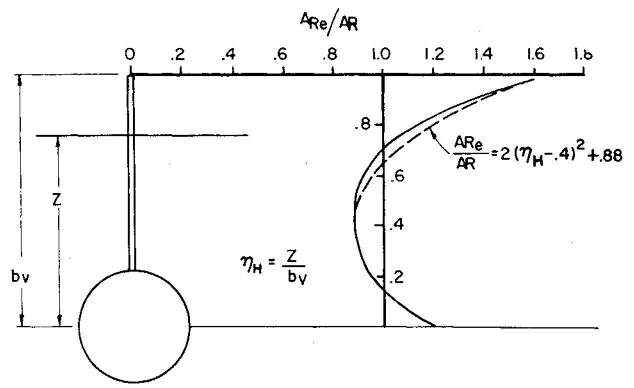

Fig. 5 Effective aspect ratio of a vertical stabilizer as influenced by horizontal stabilizer location.

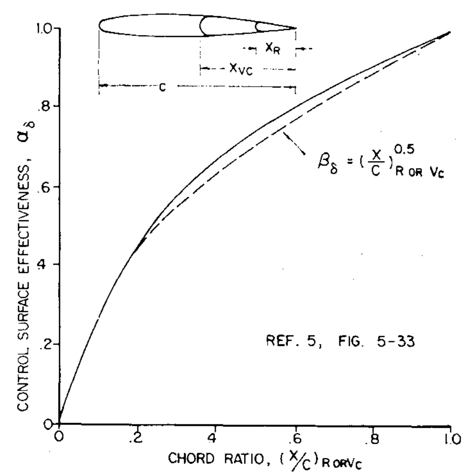

Fig. 6 Control surface effectiveness.

considered in the present study, chord ratio is conveniently taken as equal to the area ratio. The relations indicated on Fig. 4–6 have been approximated as continuous functions in expressing these factors in the computer simulation with the approximate expressions utilized indicated on each figure.

Results

The system developed using the factors and relationships described in foregoing sections has been applied to a typical STOL transport configuration to demonstrate the optimization process. Pertinent configuration fixed variables are: wing span, 113.2 ft; wing mean chord, 15.5 ft; tail length, 52.5 ft; wing area, 1600 sq. ft; airplane weight, 159,700 lbs; yawing moment, 720,000 ft/lb; tail sweep (0.5 c), 35°; and tail taper ratio, 0.3.

Portions of the results obtained using the digital computer program described in the preceding section are shown in Table 1. This particular run utilized convergence limits of 0.05 on all decision variables and reduction factors for both step size and convergence limits of 0.4. More recent analyses to define sensitivities of the factors involved indicated that convergence time could be considerably reduced by more realistic specification of these limits based on practical design applications.

The initial solution vector, composed of the six decision variables noted on Table 2 was used to establish a feasible starting point as shown in Table 2. Beginning values of both the penalty function and objective function were established: penalty function, $P = 3846$; and objective function, weight = 3221.

Table 1 Portion of output from iteration process

Iteration	Decision variable no.	Decision variable value	Penalty function value	Objective function value
16	1	30.4838	3939.89	
17	1	32.2962	3843.83	
18	1	31.3900	3839.80	
19	2	12.0100	3837.36	
34	2	41.2150	3248.49	
35	2	29.4823	3169.21	
36	2	31.7236	3166.22	
37	3	0.0100	3164.86	
49	3	0.3900	3148.70	
87	6	0.0400	1798.60	786.82

Table 2 Solution vector

Decision variable no.	Decision variable	Value initial	Value final
1	$\delta_{R\max}$	25°	31.39°
2	$\delta_{Vc\max}$	12°	31.72°
3	η_H	0	0.39
4	$(x/c)_R$	0.20	0.25
5	$(x/c)_{Vc}$	0.45	0.47
6	$\delta_{S\max}$	0°	0.04°

The initial search consists of determination of a value for the first decision variable $\delta_{R\max}$ consistent with minimization of the penalty function. Preselected step sizes are utilized in making excursions from the initial point in directions of both increasing and decreasing values of the variable. The first successful excursion (feasible point with improved penalty function) determines the next point on the course which becomes a nucleus for further search. This procedure is continued until no improved feasible point may be achieved. When the course terminates within all bounds and constraints, a nearby solution is indicated. When no advance can be produced, step sizes are reduced and the procedure repeated until step sizes fall below the prescribed level. The nucleus of probing at this point is accepted as the solution.

Several steps in the process are illustrated in Table 1. The first decision variable is varied in an increasing direction until an increase in the penalty function is produced in the seventeenth iteration. The decision variable is then reduced by smaller increments, with an attendant reduction in the value of the penalty function and the testing process continued until a maximum value is attained in the eighteenth iteration. The second variable is now introduced and a value consistent with the desired minimization of the penalty function is reached in the thirty-sixth iteration and the process continued to progressively establish values for other decision variables, progressively testing on all variables during the minimization search process. A minimum penalty, $P = 1798.6$, was produced in the eighty-seventh iteration. The associated decision variables are listed in Table 2. A structural weight of 786.82 lb is determined for the optimum configuration.

Conclusions

Application of available optimization procedures to establish the design concept for an aircraft component, based upon structural weight and performance requirements has been described. A simplified representation of the necessary system concept has indicated that results compatible with the desired goal are attainable. Extension of the method to provide for consideration of structural concepts as design variables may be readily effected. Further development of the approach utilized for application to design parametric studies would include consideration of appropriate interactions between performance, loads, stability, control, structural materials and weights requirements in configuration definition.

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